

**Homework Assignment 1****January 24, 2019**

1. Use the Euclidean Algorithm to compute the following:
  - (a)  $\gcd(1547, 560)$ .
  - (b)  $\gcd(841, 160)$ .
2. Find an inverse of 160 modulo 841.
3. Find integers  $u$  and  $v$  such that  $1547u + 560v = 7$ .
4. Find all integers  $x$  satisfying  $x^2 \equiv 2 \pmod{13}$ .
5. Show that  $x^2 + y^2 - 15z^2 = 7$  has no solutions in  $\mathbb{Z}/8\mathbb{Z}$ . Conclude that it has no solutions in  $\mathbb{Z}$ .
6. Charles de Bovelles, a French philosopher and mathematician, once claimed that, for each  $n > 1$ , at least one of  $6n + 1$  and  $6n - 1$  is a prime. Show that he was wrong.
7. de Bovelles immediately realised the nonsense and revised his claim to read that every prime, except 2 and 3, can be expressed in the form  $6n \pm 1$ , for some integer  $n$ . Show that this time he was right.
8. (Challenge) Find the units digit of  $3^{97}$ .
9. (Challenge) Let  $a, b$  and  $c$  be positive integers. Show that  $\gcd(a \cdot c, b \cdot c) = c \cdot \gcd(a, b)$  by showing that LHS divides RHS and that RHS divides LHS.