Practice Problems

1 From the book

- Section 1.3: 1.15, 1.16, 1.17, 1.19, 1.20.

- Section 1.5: 1.32 (a) (b) (c).

2 Additional problems

- Use FLT to show that 77 is not prime.

- Compute all invertible elements modulo 8, i.e. determine $(\mathbb{Z}/8\mathbb{Z})^*$.

- Prove that $(\mathbb{Z}/8\mathbb{Z})^*$ has no generator. This shows that the Primitive Root Theorem does not necessarily hold for non-prime numbers.

- Find all primitive roots modulo 13.

- Let p be a prime and $e \geq 1$. Show that $\phi(p^e) = p^e - p^{e-1}$.

- Compute $\phi(126)$.

- (Challenge) Find all positive integers n such that $\phi(n) = 4$.

- (Challenge) Let $p > 3$ be a prime and g a primitive root modulo p. Show that $(p-1)! \equiv g^{\frac{p(p-1)}{2}} \pmod{p}$. Conclude that $(p-1)! \equiv -1 \pmod{p}$.