Solutions to Practice Problems

- Show that for any integer n, $2 \mid n(n+1)$ and $3 \mid n(n+1)(n+2)$.

  Among any two consecutive integers there is an even one and so $2 \mid n(n+1)$.

  Let $n = 3q + r$ for some integers q and $0 \leq r < 3$. If $r = 0$, then $3 \mid n$. If $r = 1$, then $3 \mid n+2$. If $r = 2$, then $3 \mid n+1$. Therefore, $3 \mid n(n+1)(n+2)$.

- Show that for any integer n, $\gcd(22n+7, 33n+10) = 1$.

  Let $d = \gcd(22n+7, 33n+10)$. We have that $d \mid 22n+7$ and $d \mid 33n+10$. Therefore,

  $$d \mid 3(22n+7) - 2(33n+10) = 1$$

  and so $d = 1$.

- Can we find integers a and b such that $\gcd(a,b) = 3$ and $a + b = 65$?

  Since 65 is not a multiple of $\gcd(a,b)$, Bézout's Lemma tells us that there are no integers u and v such that $au + bv = 65$.

- Show that if x and y are odd, then $x^2 + y^2$ cannot be a square.

  Every square is congruent to either 0 or 1 modulo 4. If x and y are both odd, $x^2 + y^2 \equiv 2 \pmod 4$.

- Show that if $a \equiv b \pmod m$, then $\gcd(a,m) = \gcd(b,m)$.

  We have shown this in class.

- List all integers x in the range $1 \leq x \leq 100$ such that $x \equiv 7 \pmod{100}$.

  They are the integers of the form $100k + 7$ in the interval above and so just 7.

- Show that if n is any odd integer, then $n^2 \equiv 1 \pmod 8$.

  We have that n is either $1, 3, 5$ or 7 modulo 8. In all cases $n^2 \equiv 1 \pmod 8$.